Natural Language Processing

Softmax regression

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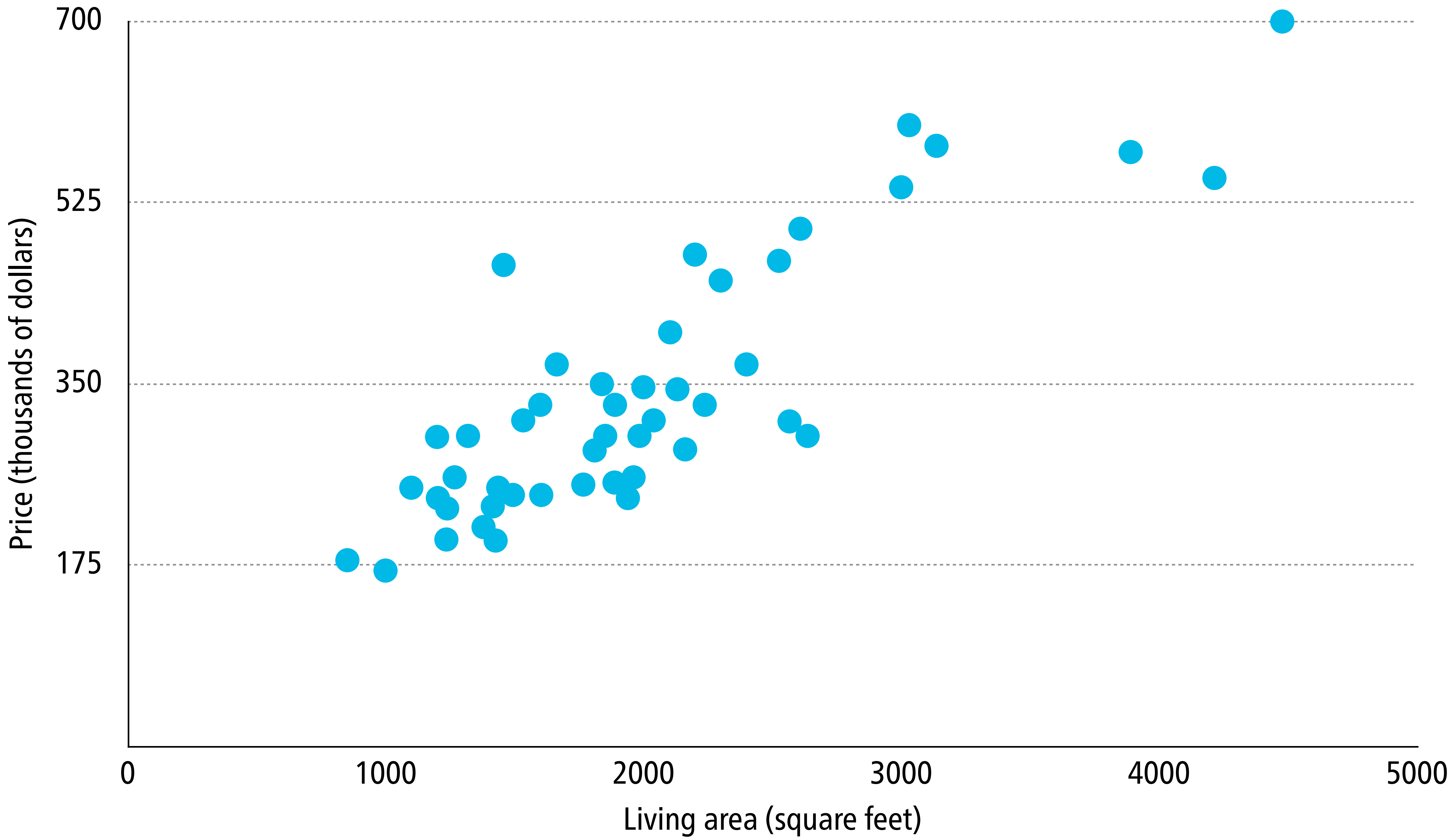
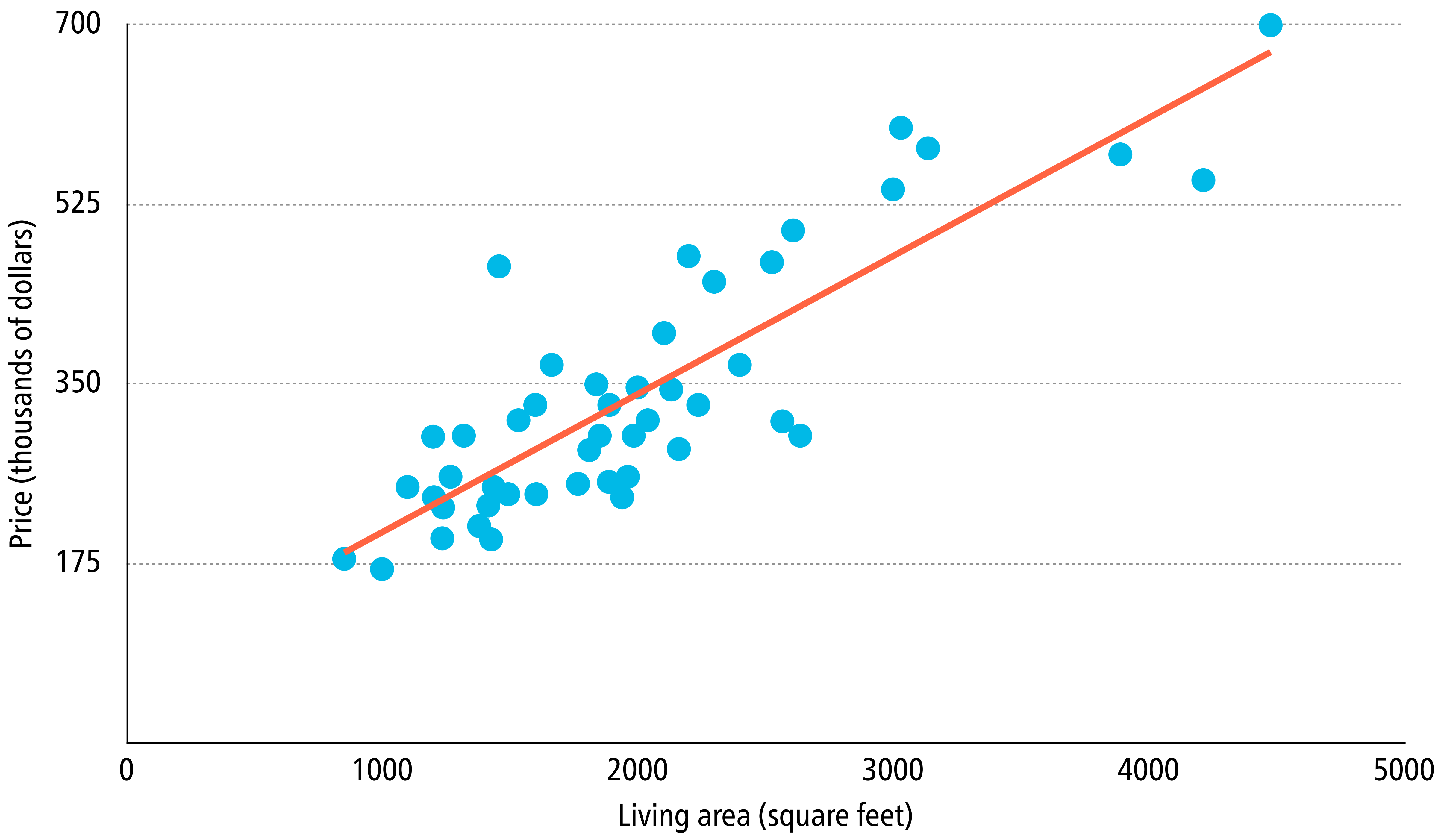
Linear regression with one variable



Linear regression with one variable



The simple linear model

input vector (1-by-F)

Convention: Input vectors are row vectors

output value

$$\hat{y} = \boldsymbol{x}\boldsymbol{w} + b$$

bias

weight vector (F-by-1)

F = number of features (independent variables)

The general linear model

input vector (1-by-m)

output vector (1-by-n)

$$\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}$$

bias vector (1-by-n)

weight matrix (m-by-n)

m = number of input features, n = number of output features

Linear classification

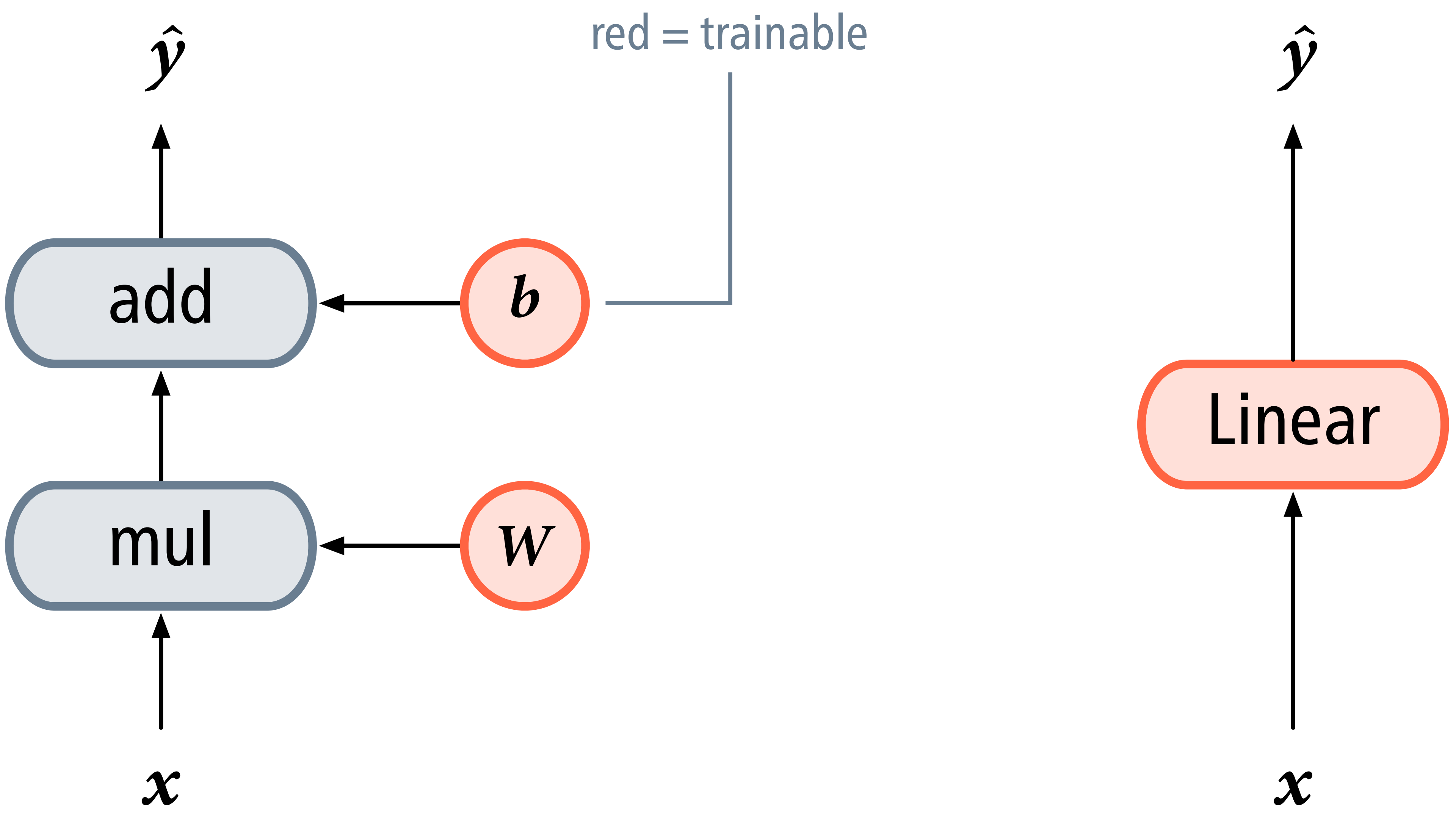
- We think of $\boldsymbol{z} = \boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}$ as an n-dimensional vector of scores that quantify the compatibility of the input $\boldsymbol{x}$ with each class k.

  higher score = higher compatibility

- In **linear classification**, we predict the input $\boldsymbol{x}$ to belong to the highest-scoring class k.

- With linear models, we can only solve a rather restricted class of classification problems (linearly separable).

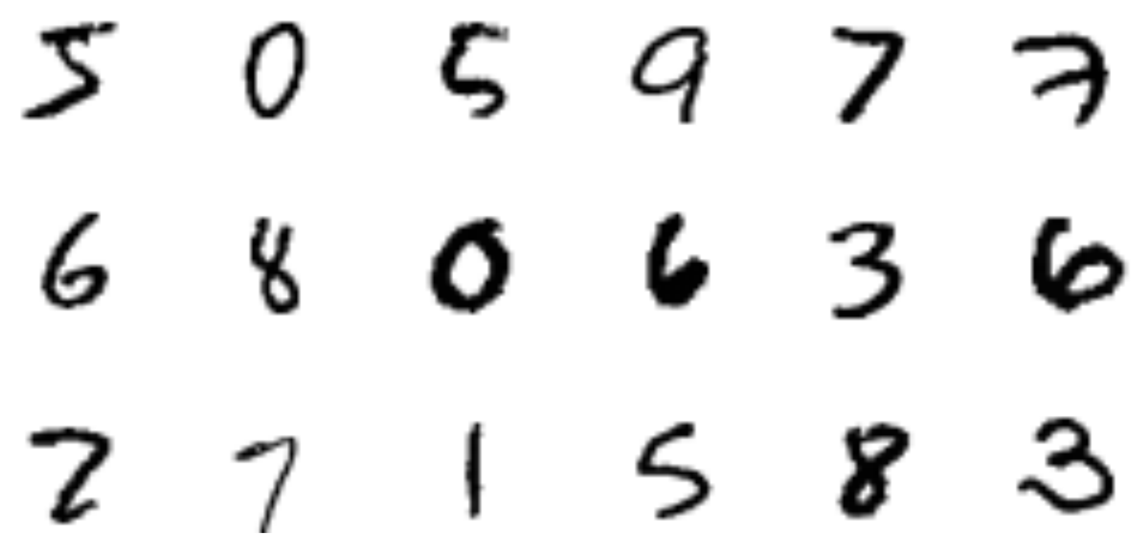
Graphical notation



computation graph

shorthand notation

Handwritten digit recognition



Input: an image of a digit, represented as a 768-dimensional vector of greyscale values.

Output: the digit depicted in the image

Softmax regression

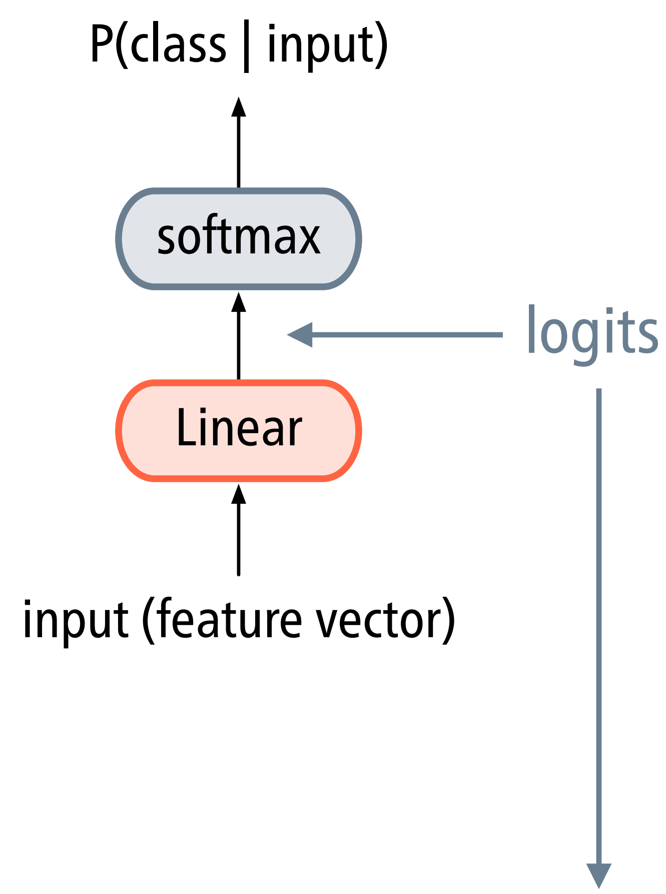
- We convert the scores into a probability distribution $P(k \mid \boldsymbol{x})$ over the classes by sending them through the **softmax function**:

$$\text{softmax}(\boldsymbol{z})[k] = \frac{\exp(\boldsymbol{z}[k])}{\sum_i \exp(\boldsymbol{z}[i])}$$

- Similar to the case of linear classification, we now predict the input $\boldsymbol{x}$ to belong to the highest-probability class k.

- In this context, the unnormalised (raw) scores are called **logits**.

Softmax regression as a neural network



$$P(k \mid \boldsymbol{x}) = \text{softmax}(\boldsymbol{x}\boldsymbol{W} + \boldsymbol{b})$$

Training a softmax regression model

- We present the model with training samples of the form ($\boldsymbol{x}$, y) where $\boldsymbol{x}$ is a feature vector and y is the gold-standard class.
- The output of the model is a vector of conditional probabilities $P(k \mid \boldsymbol{x})$ where k ranges over the possible classes.
- We want to train the model so as to maximise the likelihood of the training data under this probability distribution.

Cross-entropy loss

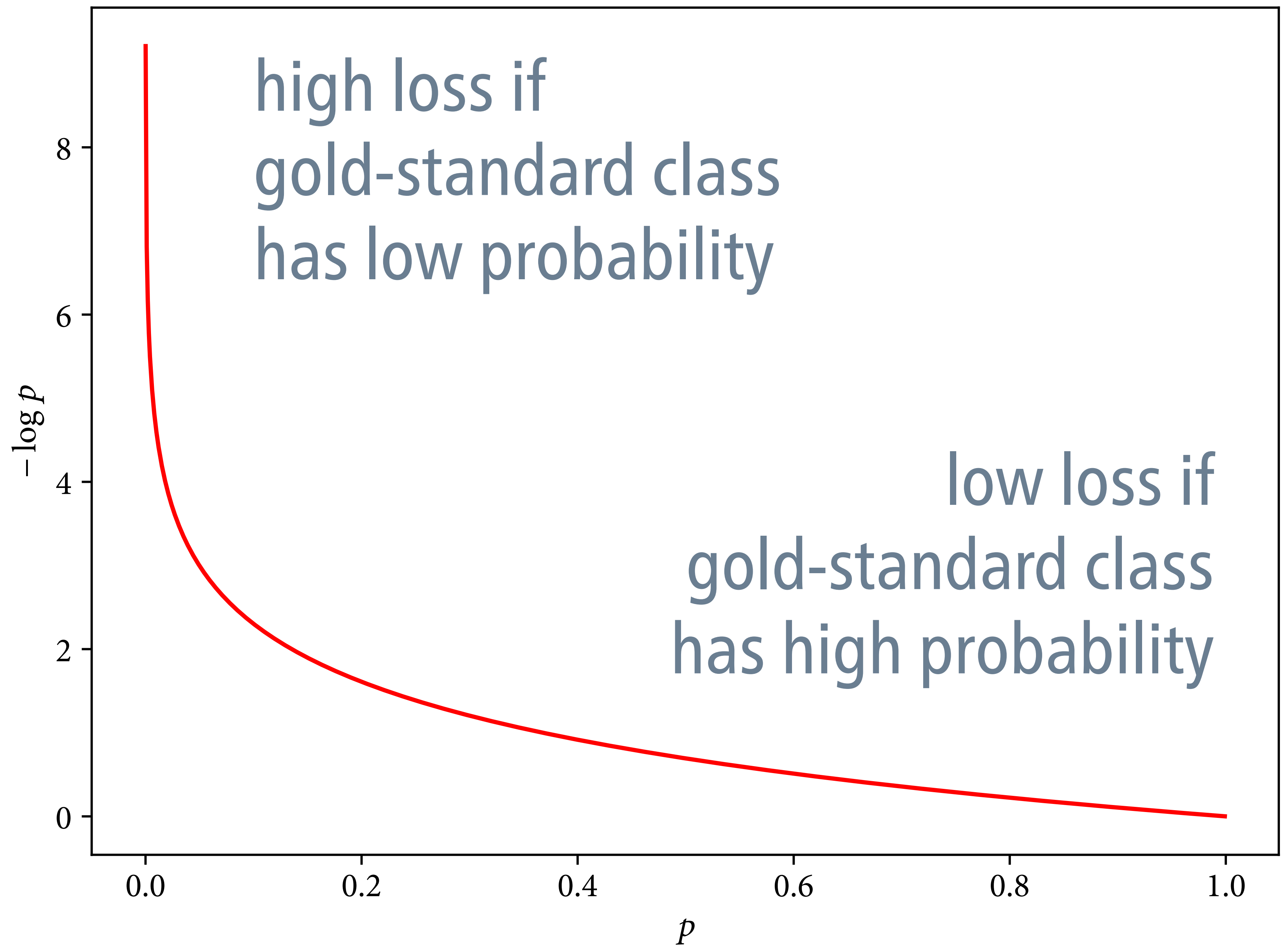
- Instead of maximising the likelihood of the training data, we minimise the model's **cross-entropy loss**.

- The cross-entropy loss for a specific sample $(\boldsymbol{x}, y)$ is the negative log probability of the gold-standard class y, in our case:

$$L(\boldsymbol{\theta}) = -\log \operatorname{softmax}(\boldsymbol{x}\boldsymbol{W} + \boldsymbol{b})[y]$$

all trainable parameters

Cross-entropy loss



Gradient descent

'Follow the gradient into the valley of error.'

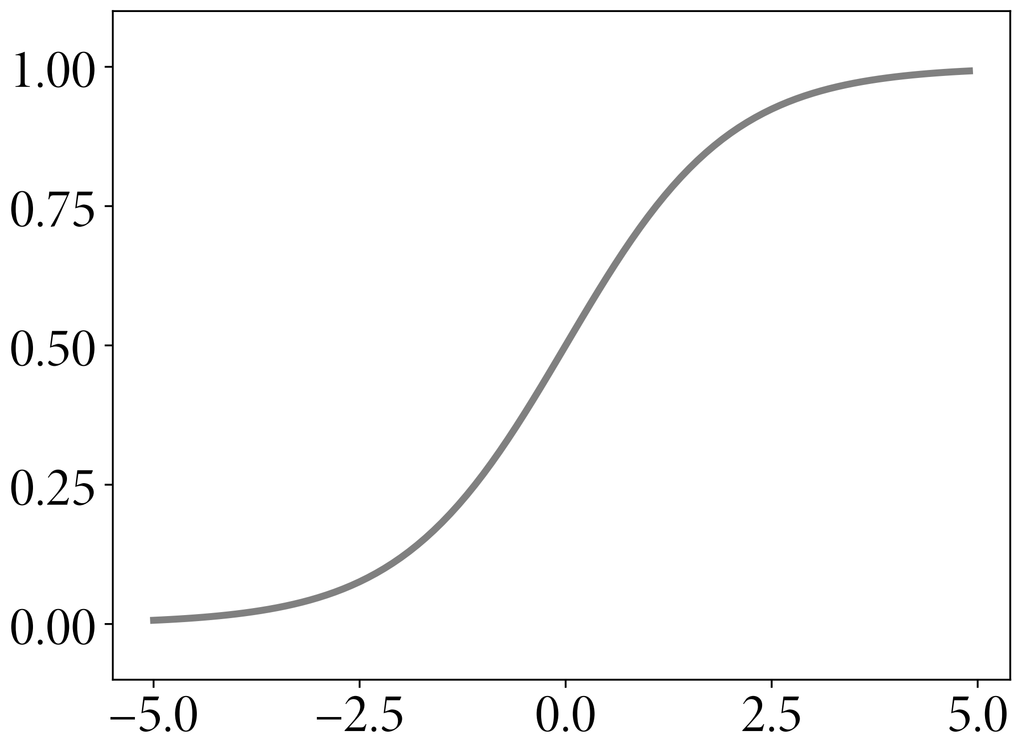
- **Step 0:** Start with random values for the parameters $\boldsymbol{\theta}$.
- **Step 1:** Compute the gradient of the loss function for the current parameter settings, $\nabla L(\boldsymbol{\theta})$.
- **Step 2:** Update the parameters $\boldsymbol{\theta}$ as follows: $\boldsymbol{\theta} \coloneqq \boldsymbol{\theta} - \alpha \, \nabla L(\boldsymbol{\theta})$

  The parameter α is the learning rate.
- Repeat step 1–2 until the loss is sufficiently low.

A note on terminology

- The softmax function can be viewed as a generalisation of the standard logistic function to more than two classes.
- What we call “softmax regression” is sometimes also called **multinomial logistic regression**.

or simply “logistic regression”



$$y = \frac{1}{1 + \exp(-z)}$$