Natural Language Processing

Lecture 2.02

Learning word embeddings via singular value decomposition

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Co-occurrence matrix

	cheese	bread	goat	sheep
cheese	14	7	5	1
bread	7	12	0	0
goat	5	0	8	12
sheep	1	0	12	2

word vector for *cheese*

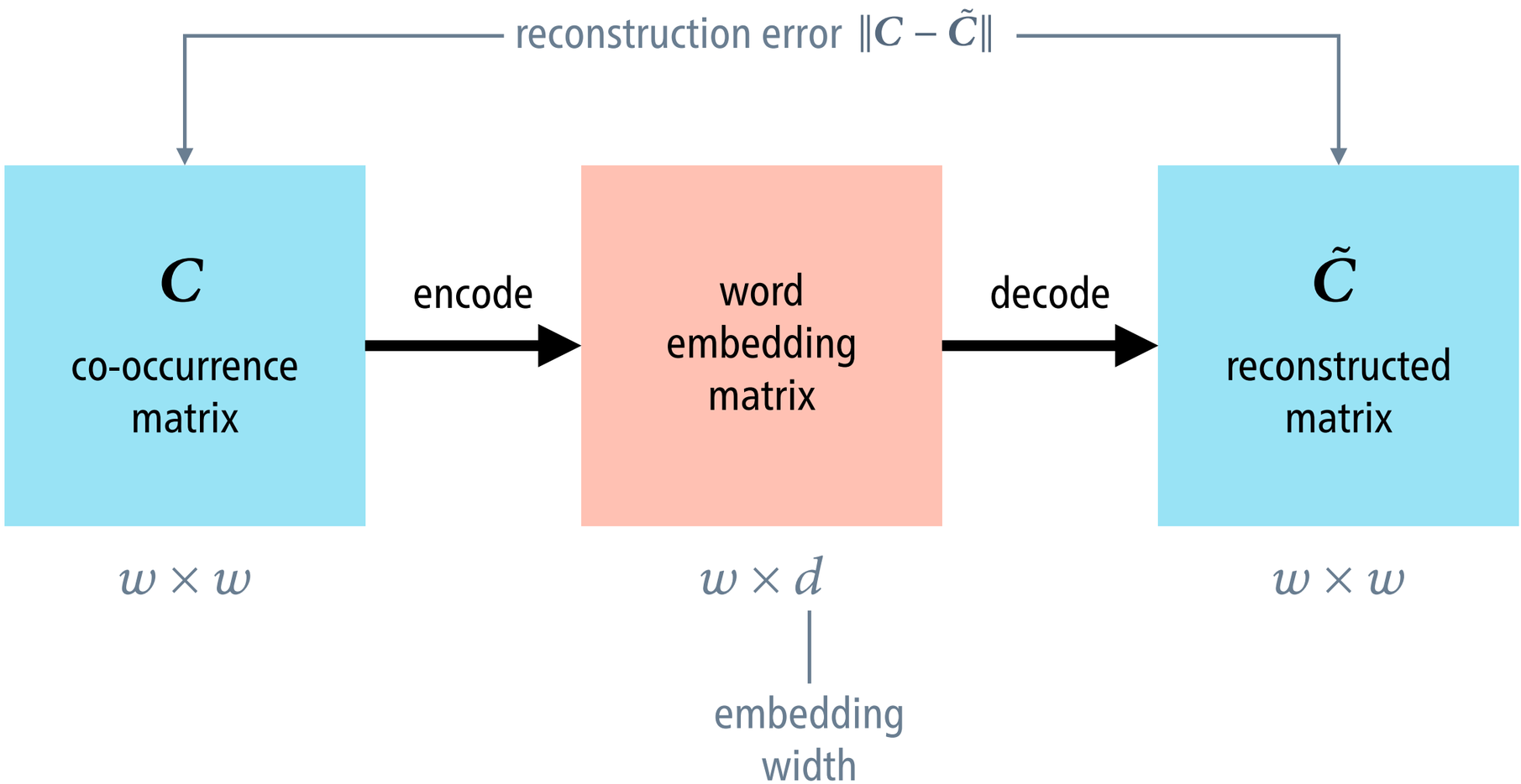
Word embeddings via dimensionality reduction

- The row vectors of co-occurrence matrices are high-dimensional, but we want word embeddings to be low-dimensional.

  hundreds instead of hundreds of thousands of dimensions

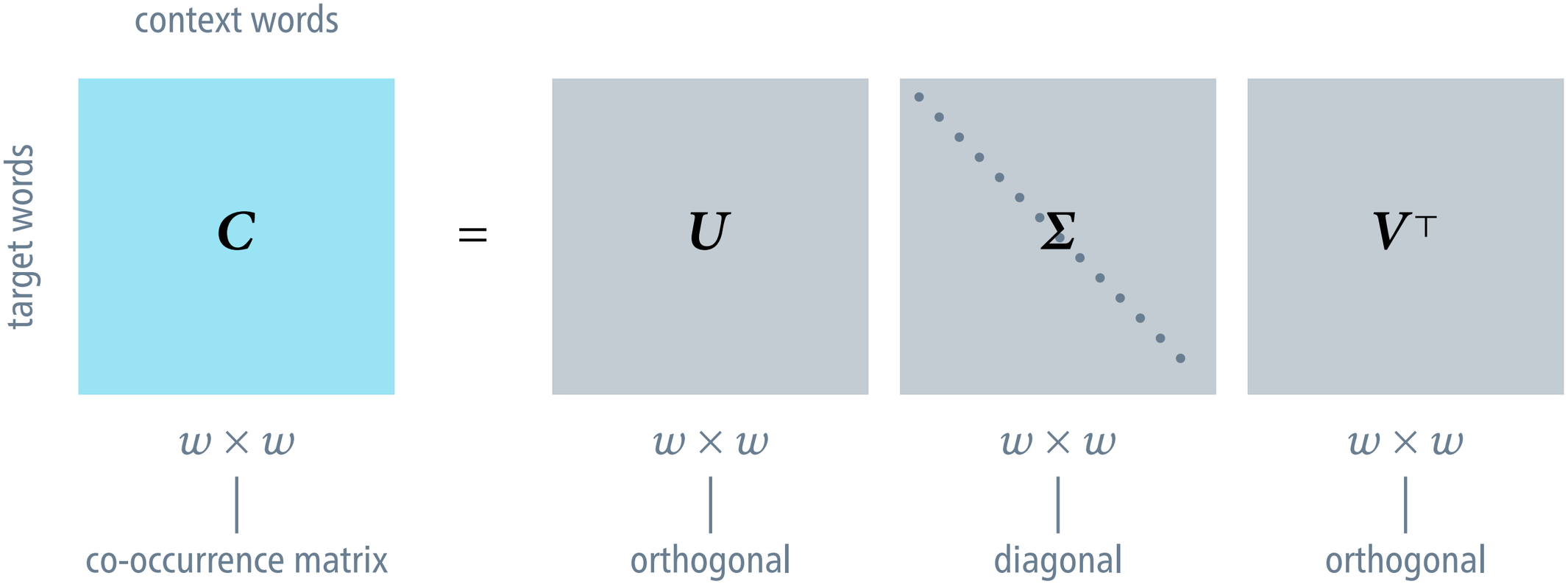
- One idea is to use dimensionality reduction and find a lower-dimensional representation of the co-occurrence matrix.

Word embeddings via dimensionality reduction



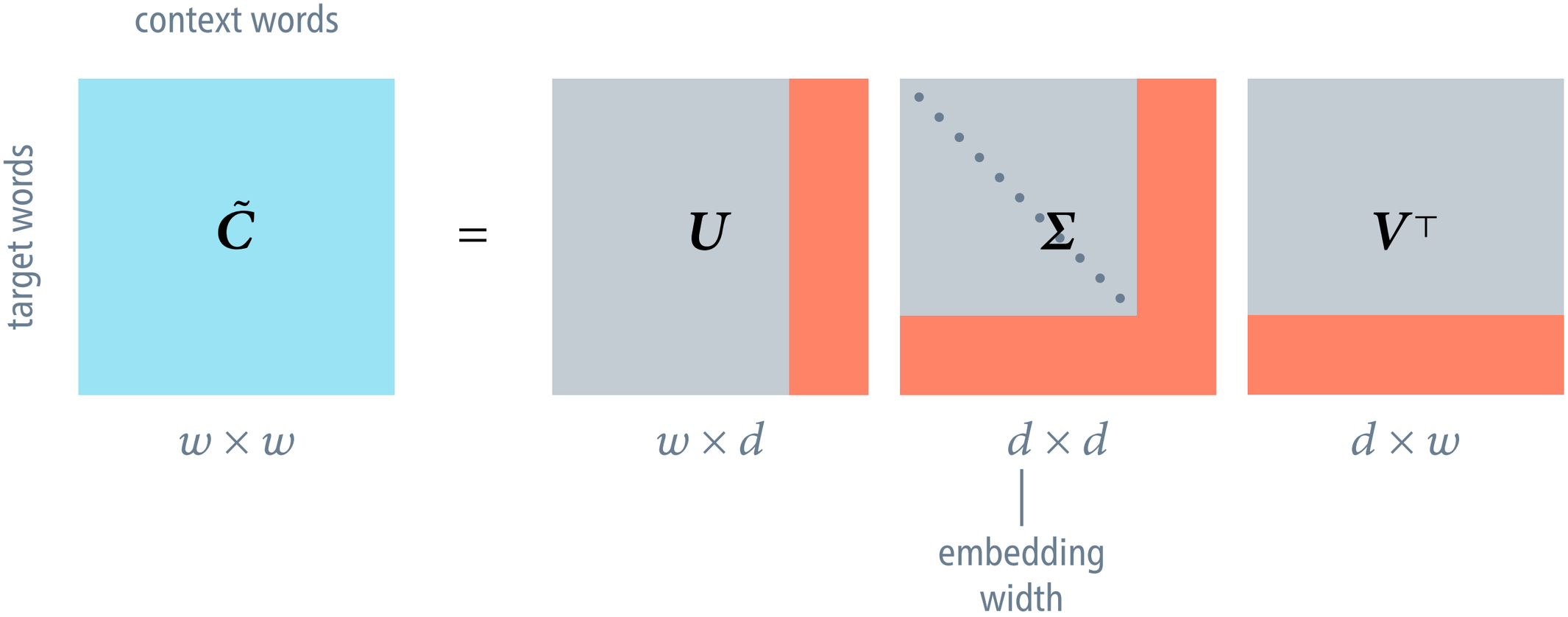
Singular value decomposition

Eisenstein § 14.3



Truncated singular value decomposition

Eisenstein § 14.3



Deerwester et al. (1990)

original co-occurrence matrix

14	7	5	1
7	12	0	0
5	0	8	12
1	0	12	2

U

–0.678	0.293	0.124	0.662
–0.445	0.562	–0.044	–0.696
–0.491	–0.591	–0.628	–0.124
–0.318	–0.499	0.767	–0.248

Σ

22.676	0.000	0.000	0.000
0.000	15.644	0.000	0.000
0.000	0.000	7.655	0.000
0.000	0.000	0.000	5.335

$V^\top$

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0.293	0.562	–0.591	–0.499
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reconstruction d = 3

11.659	9.459	5.439	1.878
9.459	9.418	–0.461	–0.922
5.439	–0.461	7.918	11.835
1.878	–0.922	11.835	1.671

reconstruction error: 1.779

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reconstruction error: 1.779

reconstruction d = 2

11.776	9.417	4.845	2.605
9.417	9.432	–0.249	–1.181
4.845	–0.249	10.934	8.148
2.605	–1.181	8.148	6.178

reconstruction error: 5.442

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2.605	–1.181	8.148	6.178

reconstruction error: 5.442

reconstruction d = 1

10.436	6.843	7.552	4.888
6.843	4.486	4.951	3.205
7.552	4.951	5.465	3.537
4.888	3.205	3.537	2.289

reconstruction error: 20.737

Truncated singular value decomposition

d = 200

d = 100

d = 50

d= 20

d = 10

d = 5

Reading off word embeddings

- We would like to quantify the similarity of word embeddings. The main operation involved in this is the dot product.

  cosine similarity = dot product of length-normalised vectors

- Conveniently, the dot products between the row vectors of the reconstructed matrix are equal to the dot products between the row vectors of the lower-dimensional matrix $\boldsymbol{U\Sigma}$.

- We could read off word embeddings from that matrix; but empirically it works better to dismiss $\boldsymbol{\Sigma}$ and only use $\boldsymbol{U}$.

Levy et al. (2015)

Limitations of the singular value decomposition

- Computing the singular value decomposition is expensive; as a rule of thumb, the running time is in $\Theta(V^2 d)$.

  In practice, randomized algorithms are used.

- The method is not incremental – the decomposition needs to be recomputed from scratch whenever new data arrives.

Mahoney (2016)

Pointwise mutual information

- Raw counts give too much weight to function words such as *the*, *she*, *has*, and too little weight to *cheese*, *bread*, *sheep*.
- We measure the associative strength between a target word w and a context c using **pointwise mutual information**:

$$\text{PMI}(w, c) = \log \frac{P(w, c)}{P(w)P(c)}$$

Positive pointwise mutual information (PPMI)

- Because the rows of co-occurrence matrices are sparse, many PMI values will be $\log 0 = -\infty$.
- A common approach is to use **positive pointwise mutual information (PPMI)**, and replace all negative values with zero:

$$\text{PPMI}(w, c) = \max\big(\text{PMI}(w, c), 0\big)$$

- A shortcoming of this approach is its bias towards infrequent events; for these, the PPMI value will be very high.